

NOTES PROPORTIONALITY & SIMILARITY

GR 12 PROOFS

THEOREM 1

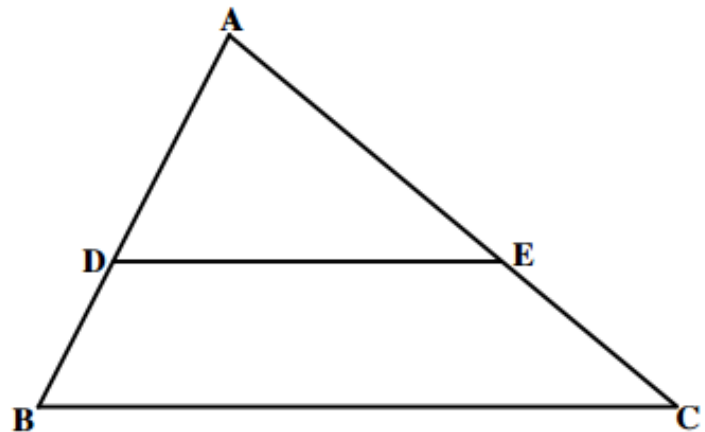
A line drawn parallel to one side of a triangle cuts the other two sides so as to divide them in the same proportion.

Given:

$$DE \parallel BC$$

Required to prove:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



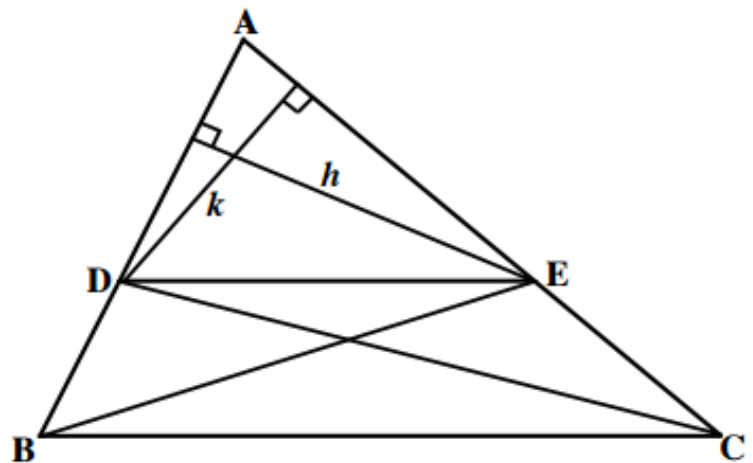
Proof:

In $\triangle ADE$, draw height h relative to base AD and height k relative to base AE .
Join BE and DC to create $\triangle BDE$ and $\triangle CED$.

$$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \cdot AD \cdot h}{\frac{1}{2} \cdot BD \cdot h} = \frac{AD}{BD}$$

$$\frac{\text{Area } \triangle ADE}{\text{Area } \triangle CED} = \frac{\frac{1}{2} \cdot AE \cdot k}{\frac{1}{2} \cdot EC \cdot k} = \frac{AE}{EC}$$

Now it is clear that
 $\text{Area } \triangle BDE = \text{Area } \triangle CED$
(same base, height and lying between parallel lines)



$$\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle CED}$$

$$\therefore \frac{AD}{BD} = \frac{AE}{EC}$$

THEOREM

THEOREM 3

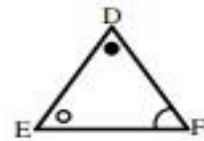
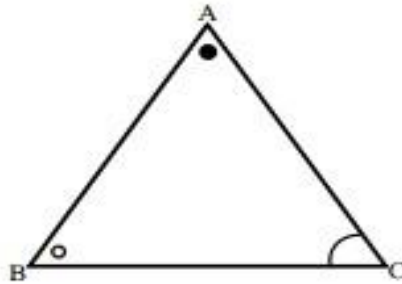
The corresponding sides of two equiangular triangles are in the same proportion and therefore the triangles are similar.

Given:

$$\hat{A} = \hat{D}, \hat{B} = \hat{E} \text{ and } \hat{C} = \hat{F}$$

Required to prove:

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$



Proof:

On AB mark off AG = DE .

On AC mark off AH = DF .

Join GH.

In $\triangle AGH$ and $\triangle DEF$:

(1) AG = DE construction

(2) $\hat{A} = \hat{D}$ given

(3) AH = DF construction

$\therefore \triangle AGH = \triangle DEF$ SAS

$$\therefore \hat{G}_1 = \hat{E}$$

But $\hat{B} = \hat{E}$ given

$$\therefore \hat{G}_1 = \hat{B}$$

$\therefore GH \parallel BC$ corresponding angles equal.

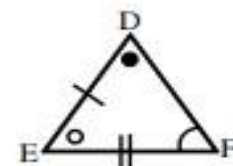
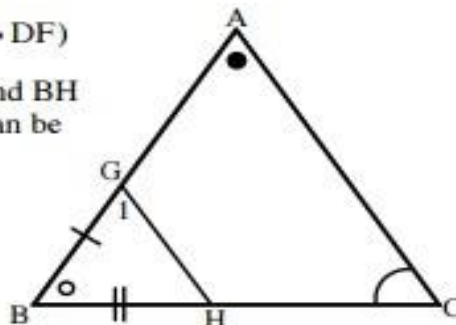
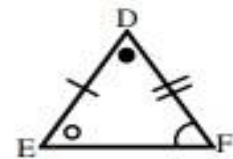
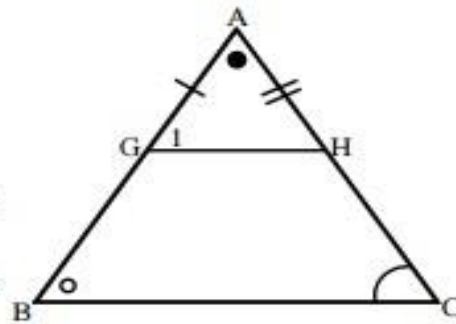
$$\therefore \frac{AB}{AG} = \frac{AC}{AH}$$

$$\therefore \frac{AB}{DE} = \frac{AC}{DF} \quad (AG = DE, AH = DF)$$

Similarly, by constructing BG and BH on AB and BC respectively, it can be proved that

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

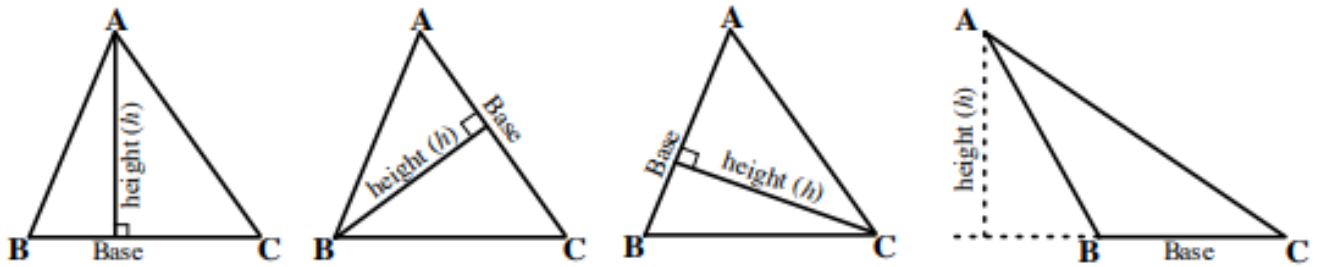


Therefore the triangles are similar.

IMPORTANT CONCEPTS REQUIRED FOR TRIANGLE GEOMETRY

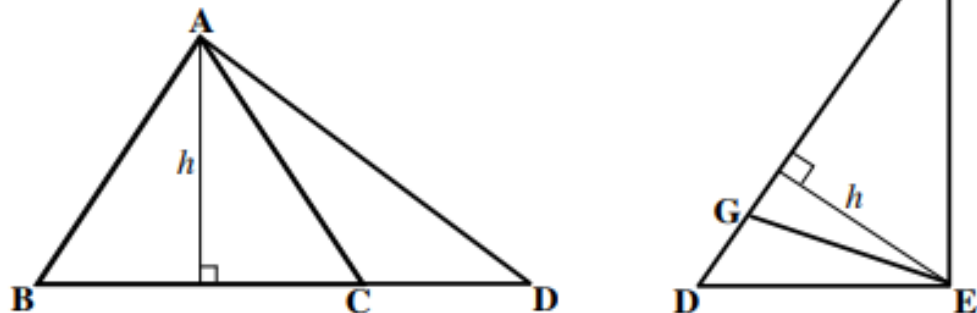
1. Area of Triangles

- (a) The height or altitude of a triangle is always relative to the chosen base.



In all cases, the area of the triangles can be calculated by using the formula $\text{Area } \triangle ABC = \frac{1}{2}(\text{base})(\text{height})$.

- (b) Two triangles which share a common vertex have a common height.



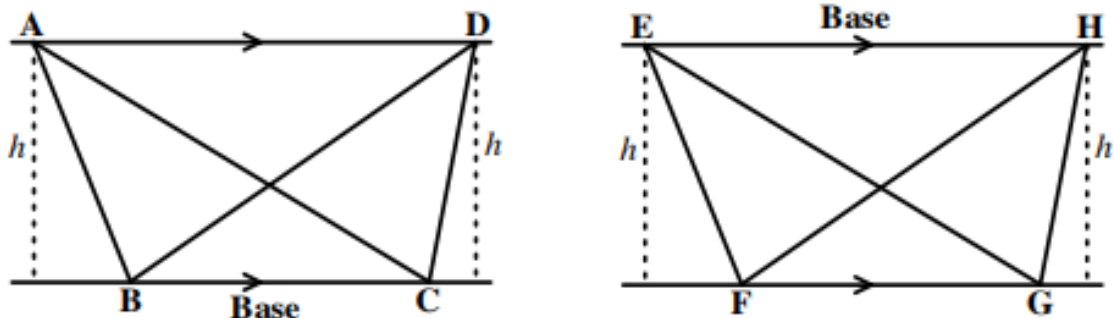
$$\text{Area } \triangle ABC = \frac{1}{2} \cdot BC \cdot h$$

$$\text{Area } \triangle ACD = \frac{1}{2} \cdot CD \cdot h$$

$$\text{Area } \triangle DEG = \frac{1}{2} \cdot DG \cdot h$$

$$\text{Area } \triangle FGE = \frac{1}{2} \cdot GF \cdot h$$

- (c) Triangles with equal or common bases lying between parallel lines have the same area.



$$\text{Area } \triangle ABC = \text{Area } \triangle DBC$$

$$\text{Area } \triangle EFH = \text{Area } \triangle HGFE$$

3. Cross multiplication techniques

Cross multiplication is useful when working with ratios.

Consider, for example, the ratios $\frac{3}{2} = \frac{6}{4}$

The following statements are true for the given ratio:

If $\frac{3}{2} = \frac{6}{4}$ then:

(a) $\frac{2}{3} = \frac{4}{6}$ (invert both left and right)

(b) $\frac{3}{6} = \frac{2}{4}$ (interchange 6 and 2)

(c) $\frac{4}{2} = \frac{6}{3}$ (interchange 3 and 4)

(d) $3 \times 4 = 6 \times 2$ (multiply 3 by 4 and 6 by 2)

(e) $\frac{3 \times 4}{2} = 6$ (multiply 3 by 4 only)

(f) $3 = \frac{6 \times 2}{4}$ (multiply 6 by 2 only)

In general, for the ratios $\frac{a}{b} = \frac{c}{d}$

If $\frac{a}{b} = \frac{c}{d}$ then:

(a) $\frac{b}{a} = \frac{d}{c}$

(b) $\frac{a}{c} = \frac{b}{d}$

(c) $\frac{d}{b} = \frac{c}{a}$

(d) $ad = bc$

(e) $\frac{ad}{b} = c$

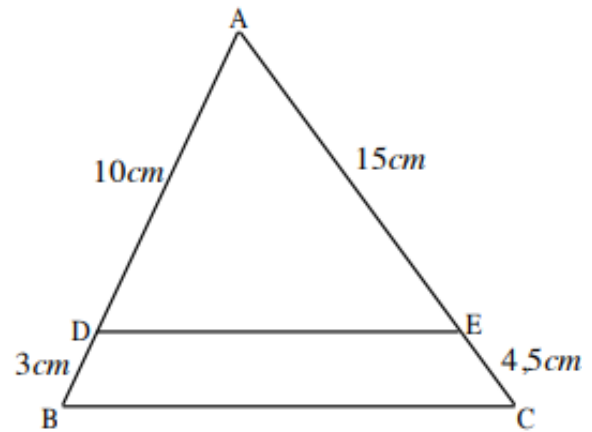
(f) $a = \frac{bc}{d}$

EXAMPLE 5

In $\triangle ABC$, $AD = 10\text{ cm}$, $DB = 3\text{ cm}$, $AE = 15\text{ cm}$ and $EC = 4,5\text{ cm}$.

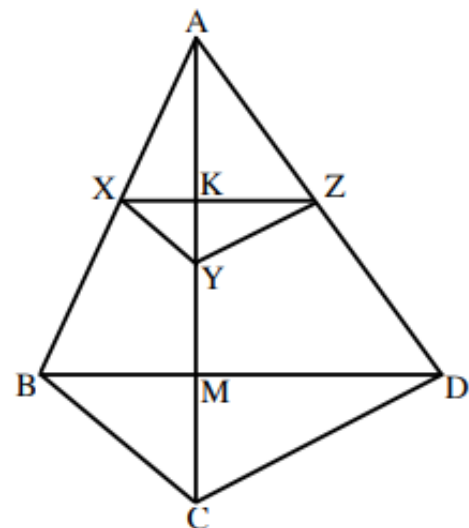
Prove that $DE \parallel BC$.

Statement	Reason
$\frac{AD}{DB} = \frac{10\text{cm}}{3\text{cm}} = \frac{10}{3}$	
$\frac{AE}{EC} = \frac{15\text{cm}}{4,5\text{cm}}$	
$\therefore \frac{AE}{EC} = \frac{15\text{cm}}{\frac{9}{2}\text{cm}}$	
$\therefore \frac{AE}{EC} = 15 \times \frac{2}{9}$	
$\therefore \frac{AE}{EC} = \frac{30}{9} = \frac{10}{3}$	
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$	
$\therefore DE \parallel BC$	Line divides sides of $\triangle ABC$ prop

**EXAMPLE 6**

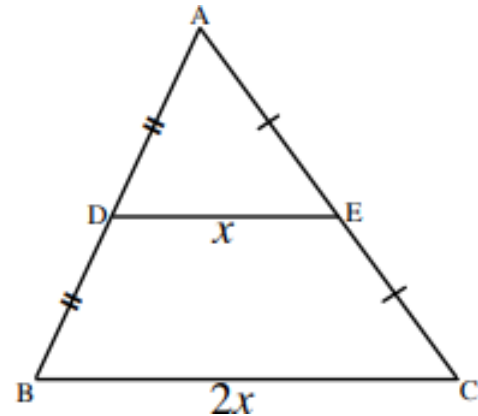
In $\triangle ABC$ and $\triangle ACD$, $XY \parallel BC$ and $YZ \parallel CD$. Prove that $XZ \parallel BD$.

Statement	Reason
In $\triangle ABC$: $\frac{AX}{XB} = \frac{AY}{YC}$	Line \parallel one side of \triangle
In $\triangle ACD$: $\frac{AZ}{ZD} = \frac{AY}{YC}$	Line \parallel one side of \triangle
$\therefore \frac{AX}{XB} = \frac{AZ}{ZD}$	Line divides sides of $\triangle ABD$ prop
$\therefore XZ \parallel BD$	



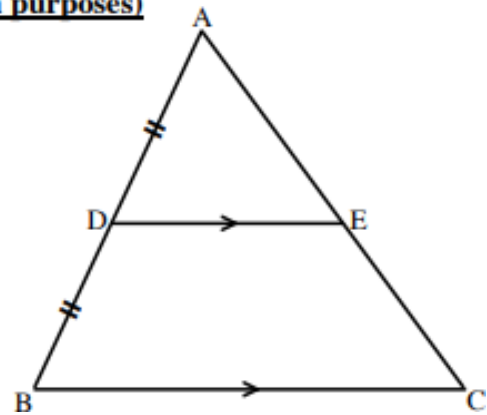
THEOREM 2 (THE MIDPOINT THEOREM)

This theorem is a special case of Theorem 1. It states that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the third side. If $AD = DB$ and $AE = EC$, then $DE \parallel BC$ and $BC = 2DE$ or $DE = \frac{1}{2}BC$.



THEOREM 2 CONVERSE (not for examination purposes)

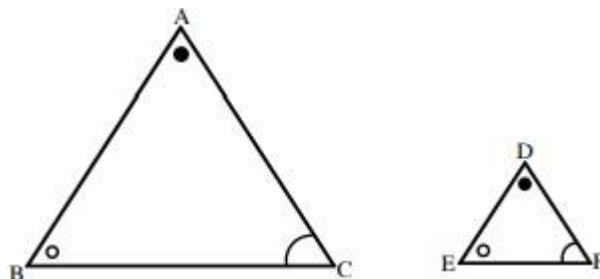
The line passing through the midpoint of one side of a triangle, parallel to another side, bisects the third side and is equal to half the length of the side it is parallel to. If $AD = DB$ and $DE \parallel BC$, then $AE = EC$ and $BC = 2DE$ or $DE = \frac{1}{2}BC$.



SIMILARITY THEOREMS

Two conditions must **both** be satisfied for two polygons to be similar:

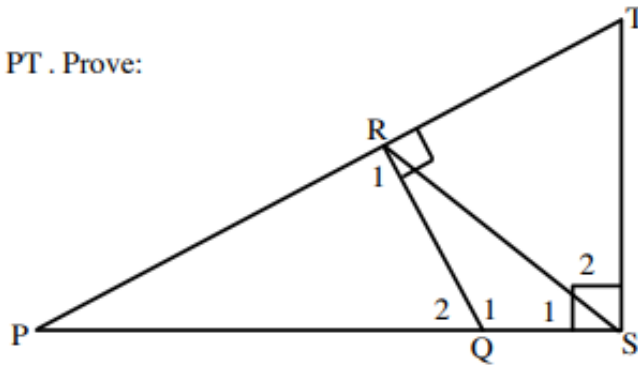
- The corresponding angles must be equal.
- The ratio of the corresponding sides must be in the same proportion.



EXAMPLE 8

In ΔPST , $TS \perp PS$ and $RQ \perp PT$. Prove:

- (a) $\Delta PRQ \parallel \Delta PST$
- (b) $RQ : PQ = ST : PT$
- (c) $PR \cdot PT = PQ \cdot PS$

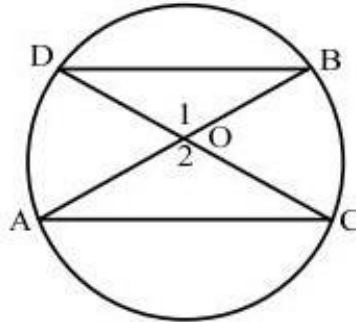


Statement	Reason
<p>(a) Match the corresponding angles of ΔPRQ and ΔPST as follows and then prove the pairs of angles equal.</p> <p>$\hat{P} \text{ --- } \hat{P}$ Draw solid lines for each pair of corresponding angles that are equal.</p> <p>$\hat{R}_1 \text{ --- } \hat{S}_1 + \hat{S}_2$ The dotted line indicates that the pair of angles are equal due to the sum of the angles of a triangle.</p> <p>$\hat{Q}_2 \text{ } \hat{T}$</p> <p>In ΔPRQ and ΔPST :</p> <p>(1) $\hat{P} = \hat{P}$</p> <p>(2) $\hat{R}_1 = \hat{S}_1 + \hat{S}_2 = 90^\circ$</p> <p>(3) $\hat{Q}_2 = \hat{T}$</p> <p>$\therefore \Delta PRQ \parallel \Delta PST$</p>	<p>common</p> <p>given</p> <p>sum of angles of Δ</p> <p>\angle, \angle, \angle</p>
<p>(b) Since $\Delta PRQ \parallel \Delta PST$:</p> <p>$\frac{PR}{PS} = \frac{RQ}{ST} = \frac{PQ}{PT}$</p> <p>$\therefore \frac{RQ}{ST} = \frac{PQ}{PT}$</p> <p>$\therefore \frac{RQ}{PQ} = \frac{ST}{PT}$</p> <p>$\therefore RQ : PQ = ST : PT$</p>	<p>corr sides of Δ's in proportion</p> <p>cross multiplication</p>
<p>(c) $\frac{PR}{PS} = \frac{PQ}{PT}$</p> <p>$\therefore PR \cdot PT = PQ \cdot PS$</p>	<p>since $\frac{PR}{PS} = \frac{RQ}{ST} = \frac{PQ}{PT}$</p> <p>cross multiplication</p>

EXAMPLE 9

A, B, C and D are concyclic points. DOC and AOB are chords. DB and AC are joined. Prove that:

- (a) $\triangle AOC \parallel \triangle DOB$
 (b) $\frac{OB}{OD} = \frac{OC}{OA}$

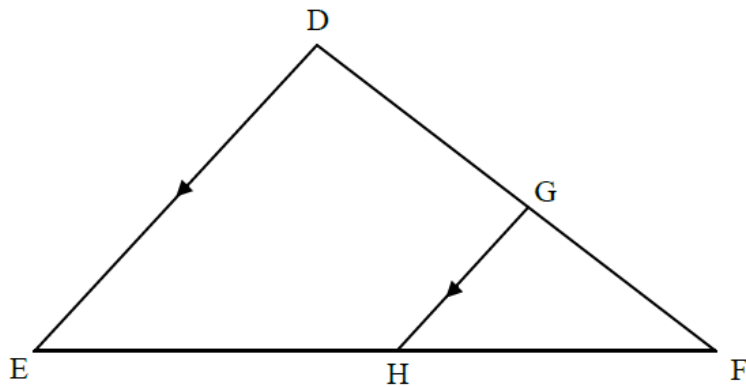


Statement	Reason
<p>(a) Match the corresponding angles of $\triangle AOC$ and $\triangle DOB$ as follows and then prove the pairs of angles equal.</p> <p>$\hat{A} \text{ --- } \hat{D}$ Draw solid lines for each pair of corresponding angles that are equal.</p> <p>$\hat{O}_2 \text{ } \hat{O}_1$ The dotted line indicates that the pair of angles are equal due to the sum of the angles of a triangle.</p> <p>$\hat{C} \text{ --- } \hat{B}$</p> <p>In $\triangle AOC$ and $\triangle DOB$:</p> <p>(1) $\hat{A} = \hat{D}$</p> <p>(2) $\hat{C} = \hat{B}$</p> <p>(3) $\hat{O}_2 = \hat{O}_1$</p> <p>$\therefore \triangle AOC \parallel \triangle DOB$</p> <p>Note: You could have also used the reason "vertically opposite angles" for statement (3) above.</p>	<p>arc BC subtends equal angles</p> <p>arc AD subtends equal angles</p> <p>sum of angles of \triangle</p> <p>\angle, \angle, \angle</p>
<p>(b) $\triangle AOC \parallel \triangle DOB$</p> <p>$\therefore \frac{AO}{DO} = \frac{OC}{OB} = \frac{AC}{DB}$</p> <p>$\therefore \frac{OA}{OD} = \frac{OC}{OB}$</p> <p>$\therefore \frac{OB}{OD} = \frac{OC}{OA}$</p>	<p>corr sides of \triangle's in proportion</p> <p>cross multiplication</p>

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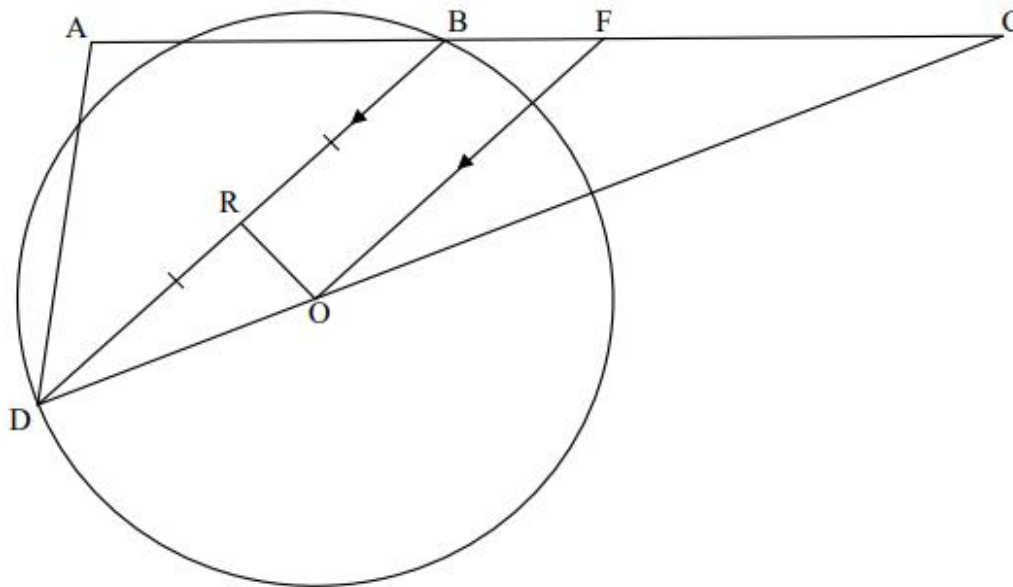
QUESTION 9

- 9.1 In the diagram, $\triangle DEF$ is drawn. Line GH intersects DF and EF at G and H respectively such that $GH \parallel DE$ and $\frac{GF}{DG} = \frac{2}{5}$.



- 9.1.1 Write down, with a reason, the value of $\frac{HF}{EH}$. (2)
- 9.1.2 If $EF = 21$ cm, calculate the length of EH . (2)
- 9.1.3 Write down a triangle which is similar to $\triangle FGH$. (1)
- 9.1.4 Hence, calculate the value of $\frac{GH}{DE}$. (2)
-

- 10.2 In the diagram, O is the centre of the circle. Points D and B lie on the circle. Points A and C lie outside the circle such that side AC of $\triangle ADC$ passes through B . F is a point on BC such that $FO \parallel BD$. $DR = RB$ and RO is drawn.



10.2.1 Prove, with reasons, that $\triangle CFO \parallel \triangle CBD$. (3)

10.2.2 If it is given that $\hat{RDO} = \hat{FCO}$, show, with reasons, that $OF \cdot CD = CO \cdot BC$ (2)

10.2.3 It is further given that $DC = 19,2$ units, $BD = 12$ units and $\frac{RO}{RD} = \frac{3}{4}$
 Prove, with reasons, that $BF = \frac{75}{16}$ (6)

10.2.4 Calculate the size of \hat{ABD} . (3)

[20]